Individuals Tiebreaker 2022-2023

Student Name:

Team Name:

Rules and Directions

1. **DO NOT FLIP OR DETACH THIS PAGE UNTIL THE TIEBREAKER BEGINS.**
2. **Congratulations** for scoring well on the Individual Round. This is the CHMMC 2022-2023 **Individuals Tiebreaker**.
3. There are **4 questions with short-answers**, to be completed in **40 minutes** or **less**.
4. **You may not collaborate with your team** on the Individuals Tiebreaker. In addition, **no other collaboration, computers, calculators, or other outside aid is permitted**.
5. You are permitted to use ruler and compass but **not** a protractor.
6. On the **back side** of this page: write your answers in the corresponding boxes.
7. Answers must be *reasonably simplified* as described in the **CHMMC Conventions** document. You will not earn credit for a correct but unsimplified answer.
8. The time limit of this test is **40 minutes**, but you may submit answers at **any time** by handing this sheet of paper to the proctor. Students will be ranked based on the number of correct answers and (among those with the same number of correct answers) their submission time.

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Problems

Problem 1. Yor and Fiona are playing a match of tennis against each other. The first player to win 6 games wins the match (while the other player loses the match). Yor has currently won 2 games, while Fiona has currently won 0 games. Each game is won by one of the two players: Yor has a probability of $\frac{2}{3}$ to win each game, while Fiona has a probability of $\frac{1}{3}$ to win each game. Then, $\frac{m}{n}$ is the probability Fiona wins the tennis match, for relatively prime integers m, n. Compute m.

Problem 2. Jonathan and Eric are standing one kilometer apart on a large, flat, empty field. Jonathan rotates an angle of $\theta = 120^\circ$ counterclockwise around Eric, then Eric moves half of the distance to Jonathan. They keep repeating the previous two movements in this order. After a very long time, their locations approach a point P on the field. What is the distance, in kilometers, from Jonathan's starting location to P?

Problem 3. Suppose that a, b, c are complex numbers with $a+b+c=0$, $|abc|=1$, $|b|=|c|$, and

$$\frac{9-\sqrt{33}}{48} \leq \cos^2\left(\arg\left(\frac{b}{a}\right)\right) \leq \frac{9+\sqrt{33}}{48}.$$

Find the maximum possible value of $|-a^6+b^6+c^6|$.

Problem 4. Let ABC be a triangle with $AB=4, BC=5, CA=6$. Triangles APB and CQA are erected outside ABC such that $AP=PB$, $\overline{AP} \perp \overline{PB}$ and $CQ=QA$, $\overline{CQ} \perp \overline{QA}$. Pick a point X uniformly at random from segment $\overline{BC}$. What is the expected value of the area of triangle PXQ?

Answer Submissions

Problem 1.	**Problem 3.**
Problem 2.	**Problem 4.**