

# CHMMC 2015 Tiebreaker Problems

November 22, 2015

**Problem 0.1.** Call a positive integer  $x$   $n$ -cube-invariant if the last  $n$  digits of  $x$  are equal to the last  $n$  digits of  $x^3$ . For example, 1 is  $n$ -cube invariant for any integer  $n$ . How many 2015-cube-invariant numbers  $x$  are there such that  $x < 10^{2015}$ ?

**Problem 0.2.** Let  $a_1 = 1, a_2 = 1$ , and for  $n \geq 2$ , let

$$a_{n+1} = \frac{1}{n}a_n + a_{n-1}$$

What is  $a_{12}$ ?

**Problem 0.3.** Define an  $n$ -digit pair cycle to be a number with  $n^2 + 1$  digits between 1 and  $n$  with every possible pair of consecutive digits. For instance, 11221 is a 2-digit pair cycle since it contains the consecutive digits 11, 12, 22, and 21. How many 3-digit pair cycles exist?

**Problem 0.4.** The following number is the product of the divisors of  $n$ .

46,656,000,000

What is  $n$ ?