CMM 2026 Integration Bee Finals

Caltech Math Meet

California Institute of Technology

January 24th, 2026

Rules

You have 2 minutes to solve the indicated integral. First solve gets a point. +C and absolute value bars around ln are not needed. Arguments to all trigonometric functions are in radians.

Integral 1

$$\int \frac{1+5\sqrt{x}}{1+x}\,dx$$

Integral 1 Answer

$$\int \frac{1+5\sqrt{x}}{1+x}\,dx = \boxed{\ln(x+1)+10\sqrt{x}-10\tan^{-1}(\sqrt{x})+C}$$

Integral 2

$$\int x(x+1)^{12}(x-1)^{12}dx$$

Integral 2 Answer

$$\int x(x+1)^{12}(x-1)^{12}dx = \boxed{\frac{(x^2-1)^{13}}{26} + C}$$

Integral 3

$$\int_0^\infty x^4 e^{-\sqrt{x}} dx$$

Integral 3 Answer

$$\int_0^\infty x^4 e^{-\sqrt{x}}\, dx = \boxed{2(9!)}$$

Integral 4

$$\int_0^2 \arctan(\tan(x))dx$$

Integral 4 Answer

$$\int_0^2 \arctan(\tan(x))dx = \boxed{\frac{(\pi-2)^2}{2}}$$

Integral 5

$$\int_{-5}^{5} \frac{x^2}{1 - x + \sqrt{1 + x^2}} dx$$

Integral 5 Answer

$$\int_{-5}^{5} \frac{x^2}{1 - x + \sqrt{1 + x^2}} dx = \boxed{125/3}$$

Integral 6

$$\int_0^{\frac{2\pi}{3}} \frac{1}{2+\cos x}dx$$

Integral 6 Answer

$$\int_0^{\frac{2\pi}{3}} \frac{1}{2+\cos x}\,dx = \boxed{\frac{\pi\sqrt{3}}{6}}$$

Integral 7

$$\int_0^1 \sin(\cos^{-1} x - \sin^{-1} x) dx$$

Integral 7 Answer

$$\int_0^1 \sin(\cos^{-1} x - \sin^{-1} x) dx = \boxed{\frac{1}{3}}$$

Integral 8

$$\int_0^{36} \gcd\left(\lceil x \rceil, \left\lceil \frac{x}{6} \right\rceil\right) dx$$

Integral 8 Answer

$$\int_0^{36} \gcd\left(\lceil x \rceil, \left\lceil \frac{x}{6} \right\rceil\right) dx = \boxed{67}$$

Integral 9

$$\int \frac{dx}{\sin(x)\cos(x)\ln(\tan(x))}$$

Integral 9 Answer

$$\int \frac{dx}{\sin(x)\cos(x)\ln(\tan(x))} = \boxed{\ln(\ln(\tan(x))) + C}$$

Integral 10

$$\int_0^{100} \left\lceil \left\lceil \frac{100}{\lceil x \rceil} \right\rceil - \frac{100}{\lceil x \rceil} \right\rceil dx$$

Integral 10 Answer

$$\int_0^{100} \left\lfloor \left\lceil \frac{100}{\lceil x \rceil} \right\rceil - \frac{100}{\lceil x \rceil} \right\rfloor dx = \boxed{91}$$

Integral 11

$$\int_0^{\frac{1}{2}} \left((1+x^2)(1+x^4)(1+x^8)(1+x^{16})\dots\right) dx$$

Integral 11 Answer

$$\int_0^{\frac{1}{2}} \left((1+x^2)(1+x^4)(1+x^8)(1+x^{16})\dots\right) dx = \boxed{\frac{\ln 3}{2}}$$

Integral 12

$$\int_1^\infty \frac{dx}{x\sqrt{x^2+x}}$$

Integral 12 Answer

$$\int_1^\infty \frac{dx}{x\sqrt{x^2+x}} = \boxed{2\sqrt{2}-2}$$

Integral 13

$$\int_{-\pi}^{\pi} \frac{x^2}{1+e^{-x}}\,dx$$

Integral 13 Answer

$$\int_{-\pi}^{\pi} \frac{x^2 e^x}{1+e^x}\, dx = \boxed{\pi^3/3}$$

Integral 14

$$\int e^{e^x+x}\sin(2e^x)dx$$

Integral 14 Answer

$$\int e^{e^x+x}\sin(2e^x)dx = \boxed{e^{e^x}\left(\frac{\sin(2e^x)-2\cos(2e^x)}{5}\right)+C}$$

Integral 15

$$\int \frac{x^2 + \ln x}{\sqrt{x^4 + (x \ln x)^2}} dx$$

Integral 15 Answer

$$\int \frac{x^2 + \ln x}{\sqrt{x^4 + (x \ln x)^2}} dx = \boxed{\sqrt{\ln^2(x) + x^2} + C}$$

Integral 16

$$\int_0^{2\pi} \left(\frac{\sin(7x)}{\sin(x)} \right)^3 dx$$

Integral 16 Answer

$$\int_0^{2\pi} \left(\frac{\sin(7x)}{\sin(x)}\right)^3 dx = \boxed{74\pi}$$

Integral 17

$$\int_1^{11} \lfloor x \rfloor \left(\frac{(\lfloor x \rfloor - 1)! + 1}{\lfloor x \rfloor} - \left\lfloor \frac{(\lfloor x \rfloor - 1)! + 1}{\lfloor x \rfloor} \right\rfloor \right) dx$$

Integral 17 Answer

$$\int_1^{11} \lfloor x \rfloor \left(\frac{(\lfloor x \rfloor - 1)! + 1}{\lfloor x \rfloor} - \left\lfloor \frac{(\lfloor x \rfloor - 1)! + 1}{\lfloor x \rfloor} \right\rfloor \right) dx = \boxed{7}$$

Integral 18.5

$$\int_{-1}^{1} \frac{1}{1+e^x}\,dx$$

Integral 18.5

$$\int_{-1}^{1} \frac{1}{1+e^x}\,dx = \boxed{1}$$

Integral 18

$$\int \sin(x)\left(\ln(x) - \frac{1}{x^2}\right)dx$$

Integral 18 Answer

$$\int \sin(x)\left(\ln(x) - \frac{1}{x^2}\right)dx = \boxed{\frac{\sin(x)}{x} - \cos(x)\ln(x) + C}$$

Integral 19

$$\int \sin\left(\cos^2(x)\right) \sin\left(\sin^2(x)\right) \sin(2x)\, dx$$

Integral 19 Answer

$$\int \sin\left(\cos^2(x)\right) \sin\left(\sin^2(x)\right) \sin(2x)\, dx = \boxed{\frac{\cos(1)\cos(2x) - \sin(\cos(2x))}{4} + C}$$

Integral 20

$$\int_0^{e^4} \ln(x^{1/4} + x^{3/4})\,dx$$

Integral 20 Answer

$$\int_0^{e^4} \ln(x^{1/4} + x^{3/4})\,dx = \boxed{(e^4 - 1)\ln(e^2 + 1) + \frac{e^4}{4} + e^2}$$

Integral 21

$$\int_0^1 x^2 e^{-\lfloor \ln(x) \rfloor} dx$$

Integral 21 Answer

$$\int_0^1 x^2 e^{-\lfloor \ln(x) \rfloor} dx = \boxed{\frac{e^1 - e^{-2}}{3\left(1 - e^{-2}\right)}}$$

Integral 22

$$\int \frac{\cos(x) - \sin(x)}{1 + \cos(x)\sin(x)} dx$$

Integral 22 Answer

$$\int \frac{\cos(x) - \sin(x)}{1 + \cos(x)\sin(x)} dx = \boxed{2\tan^{-1}(\sin(x) + \cos(x)) + C}$$

Integral 23

$$\int_0^\infty \left(\ln(e^x + e^{-x}) - x\right) dx$$

Integral 23 Answer

$$\int_0^\infty \left(\ln(e^x + e^{-x}) - x\right) dx = \boxed{\frac{\pi^2}{24}}$$

Integral 24

$$\int_0^{\frac{1}{2}} \frac{1}{(1+x)^{\frac{1}{2}}(1-x)^{\frac{3}{2}}}\, dx$$

Integral 24 Answer

$$\int_0^{\frac{1}{2}} \frac{1}{(1+x)^{\frac{1}{2}}(1-x)^{\frac{3}{2}}}\, dx = \sqrt{3}-1 = \boxed{\sqrt{3}-1}$$

Integral 25

$$\int ((\ln x)^x \ln(\ln x) + (\ln x)^{x-1}) dx$$

Integral 25 Answer

$$\int ((\ln x)^x \ln(\ln x) + (\ln x)^{x-1})dx = \boxed{(\ln x)^x + C}$$

Integral 26

$$\int_1^{\infty} \left(\frac{1}{2} - \frac{\lfloor x \rfloor}{\lfloor 2x \rfloor} \right)^2 dx$$

Integral 26 Answer

$$\int_1^\infty \left(\frac{1}{2} - \frac{\lfloor x \rfloor}{\lfloor 2x \rfloor}\right)^2 dx = \boxed{\frac{\pi^2}{64} - \frac{1}{8}}$$

Integral 27

$$\int_{-\frac{1}{2}}^{\infty} \frac{\ln(x^2+x+1)}{x^2+x+1}\,dx$$

Integral 27 Answer

$$\int_{-\frac{1}{2}}^{\infty} \frac{\ln(x^2+x+1)}{x^2+x+1}\,dx = \boxed{\pi\frac{\ln(3)}{\sqrt{3}}}$$

Integral -

$$\int \frac{x^2}{x^2+1}\,dx$$

Integral - Answer

$$\int \frac{x^2}{x^2+1}\,dx = \boxed{x - \arctan x}$$

Integral -

$$\int_0^2 \sqrt{4-x^2}\,dx$$

Integral - Answer

$$\int_0^2 \sqrt{4-x^2}dx = \boxed{\pi}$$

Integral -

$$\int_{-\pi}^{\pi} \sin^4(x)\,dx$$

Integral - Answer

$$\int_{-\pi}^{\pi} \sin^4(x)dx = \boxed{\frac{3\pi}{4}}$$

Integral -

$$\int x \arctan x \, dx$$

Integral - Answer

$$\int x \arctan x \, dx = \boxed{\frac{(x^2+1)\arctan x - x}{2}}$$

Integral -

$$\int \frac{1}{x^2(x^2+1)}\,dx$$

Integral - Answer

$$\int \frac{1}{x^2(x^2+1)}\,dx = \boxed{-\frac{1}{x} - \arctan x}$$

Integral -

$$\int \sqrt{\frac{\sin x \cos x \tan x}{\sec x \csc x \cot x}}\, dx$$

Integral - Answer

$$\int \sqrt{\frac{\sin x \cos x \tan x}{\sec x \csc x \cot x}}\, dx = \boxed{\frac{2x - \sin 2x}{4}}$$

Integral -

$$\int_0^\infty x^9 e^{-x} dx$$

Integral - Answer

$$\int_0^\infty x^9 e^{-x}\, dx = \boxed{9!}$$

Integral -

$$\int_0^{\frac{\pi}{2}} \ln(\sin x) dx$$

Integral - Answer

$$\int_0^{\frac{\pi}{2}} \ln(\sin x)\,dx = \boxed{\frac{-\pi \ln 2}{2}}$$

Integral 28

$$\int_0^\infty \frac{\arctan(x^{2026})}{(1+x)^2}\,dx$$

Integral 28 Answer

$$\int_0^\infty \frac{\arctan(x^{2026})}{(1+x)^2}\,dx = \boxed{\frac{\pi}{4}}$$

Integral 29

$$\int \frac{1 - x\ln(x)}{x(e^x + \ln(x))} dx$$

Integral 29 Answer

$$\int \frac{1 - x\ln(x)}{x(e^x + \ln(x))}dx = \boxed{\ln(\ln(x) + e^x) - x + C}$$

Integral 30

$$\int_0^1 \left\lfloor \sqrt{1+\frac{1}{x}} \right\rfloor dx$$

Integral 30 Answer

$$\int_0^1 \left\lfloor \sqrt{1+\frac{1}{x}} \right\rfloor dx = \boxed{\frac{7}{4}}$$

Integral 31

$$\int \frac{\left(3x^2+1\right)\ln\left(x+\frac{1}{x}\right)}{x^2}dx$$

Integral 31 Answer

$$\int \frac{\left(3x^2+1\right)\ln\left(x+\frac{1}{x}\right)}{x^2}dx = \boxed{\frac{\left(3x^2-1\right)\ln(\frac{x^2+1}{x})+1}{x} + 8\tan^{-1}(x) - 3x + C}$$

Integral 32

$$\int_0^\infty \frac{dx}{x^4+1}$$

Integral 32 Answer

$$\int_0^\infty \frac{dx}{x^4+1} = \boxed{\frac{\pi\sqrt{2}}{4}}$$

Integral 33

$$\int_{-2}^{2} \left(\left(\left(\left((x^2-2)^2 - 2 \right)^2 - 2 \right)^2 - 2 \right)^2 - 2 \right)^2 dx$$

Integral 33 Answer

$$\int_{-2}^{2} \left(\left(\left(\left((x^2-2)^2 - 2 \right)^2 - 2 \right)^2 - 2 \right)^2 - 2 \right)^2 dx = \boxed{8 - \frac{8}{4^6 - 1}}$$

Integral 34

$$\int \frac{e^x \cos x - e^x \sin x + \cos x}{(e^x + 1)^2} dx$$

Integral 34 Answer

$$\int \frac{e^x \cos x - e^x \sin x + \cos x}{(e^x + 1)^2} dx = \boxed{\frac{\sin x}{e^x + 1} + C}$$

Integral 35

$$\int \frac{e^x + e^{3x}}{1 - e^{2x} + e^{4x}} dx$$

Integral 35 Answer

$$\int \frac{e^x + e^{3x}}{1 - e^{2x} + e^{4x}} dx = \boxed{\tan^{-1}(e^x - e^{-x}) + C}$$

Integral 36

$$\lim_{k\to\infty}\left[k\int_{-\infty}^{0}e^{kx}\arctan(e^{x})dx\right]$$

Integral 36 Answer

$$\lim_{k\to\infty}\left[k\int_{-\infty}^{0} e^{kx}\arctan(e^x)dx\right] = \boxed{\pi/4}$$

Integral 37

$$\int \ln(1+x^2)dx$$

Integral 37 Answer

$$\int \ln(1+x^2)dx = \boxed{x\ln(1+x^2) - 2x + 2\tan^{-1}(x) + C}$$

Integral 38

$$\int_0^{\pi} \left\lfloor \cos^2 \left(\frac{\pi}{4} \sum_{n=1}^{100} \lceil \sin(2^n x) \rceil \right) \right\rfloor dx$$

Integral 38 Answer

$$\int_0^{\pi} \left\lfloor \cos^2\left(\frac{\pi}{4}\sum_{n=1}^{100}\lceil \sin(2^n x)\rceil\right)\right\rfloor dx = \boxed{\pi\left(\frac{1}{4}-\frac{1}{2^{51}}\right)}$$

Integral 39

$$\int_{-1}^{0} \cos\left(\log_2(3 \cdot 2^x - 1)\right) dx$$

Integral 39 Answer

$$\int_{-1}^{0} \cos\left(\log_2(3 \cdot 2^x - 1)\right) dx = \boxed{\sin(1)}$$

Integral 40

$$\int \frac{\ln(\ln(x))^2}{x} dx$$

Integral 40 Answer

$$\int \frac{\ln(\ln(x))^2}{x}\,dx = \boxed{\ln(x)\ln(\ln(x))^2 - 2\ln(x)\ln(\ln(x)) + 2\ln(x) + C}$$

Integral 41

$$\int_0^1 \left| \frac{\lfloor 101x \rfloor}{101} - \frac{\lfloor 100x \rfloor}{100} \right| dx$$

Integral 41 Answer

$$\int_0^1 \left| \frac{\lfloor 101x \rfloor}{101} - \frac{\lfloor 100x \rfloor}{100} \right| dx = \boxed{\frac{67}{20200}}$$

Integral 42

$$\int x^{x+1}(\ln(x) + \ln^2(x))dx$$

Integral 42 Answer

$$\int x^{x+1}(\ln(x)+\ln^2(x))dx = \boxed{x^x(x\ln(x)-1)+C}$$

Integral 43

$$\int e^{x^e} x^e (ex^e + e + 1) dx$$

Integral 43 Answer

$$\int e^{x^e} x^e (ex^e + e + 1) dx = \boxed{x^{e+1} e^{x^e} + C}$$

Integral 44

$$\int \frac{(1+\sqrt[29]{x})^{28}}{x^2}dx$$

Integral 44 Answer

$$\int \frac{(1+\sqrt[29]{x})^{28}}{x^2}dx = \boxed{-\frac{(1+\sqrt[29]{x})^{29}}{x}+C}$$

Integral 45

$$\int \frac{x^{2x}\left(1+\ln\left(x\right)\right)-x^{x}-\ln\left(x\right)}{x^{x}-1}dx$$

Integral 45 Answer

$$\int \frac{x^{2x}\left(1+\ln\left(x\right)\right)-x^{x}-\ln\left(x\right)}{x^{x}-1}dx = \boxed{x^{x}+x\ln(x)-x+C}$$

Integral 46

$$\int_0^{\frac{\pi}{2}} \prod_{n=2}^{\infty} \frac{\cot\left(\frac{\pi}{n}\right) - \cot x}{\left|\cot\left(\frac{\pi}{n}\right) - \cot x\right|} dx$$

Integral 46 Answer

$$\int_0^{\frac{\pi}{2}} \prod_{n=2}^{\infty} \frac{\cot\left(\frac{\pi}{n}\right) - \cot x}{\left|\cot\left(\frac{\pi}{n}\right) - \cot x\right|} dx = \boxed{2\pi \ln 2 - \frac{3\pi}{2}}$$

Integral 47

$$\int_0^1 (x^2 - 1)^k \, dx$$

Integral 47 Answer

$$\int_0^1 (x^2-1)^k \, dx = \boxed{(-1)^k \frac{2^{2k}(k!)^2}{(2k+1)!}}$$

Integral 48

$$\int_0^{\frac{\pi}{12}} \frac{\cos x + \sin x}{\cos^3 x - \sin^3 x}\, dx$$

Integral 48 Answer

$$\int_0^{\frac{\pi}{12}} \frac{\cos x + \sin x}{\cos^3 x - \sin^3 x}\, dx = \boxed{\frac{\ln(5/2)}{3}}$$

Integral 49

$$\int_0^1 \frac{x}{\sqrt{-\ln(x)}} dx$$

Integral 49 Answer

$$\int_0^1 \frac{x}{\sqrt{-\ln(x)}} dx = \boxed{\sqrt{\frac{\pi}{2}}}$$

Integral 50

$$\int_0^{100} \left(10 \left\lfloor (10x)^{\frac{2}{3}} \right\rfloor + \lfloor x \rfloor^{\frac{3}{2}}\right) dx$$

Integral 50 Answer

$$\int_0^{100} \left(10 \left\lfloor (10x)^{\frac{2}{3}} \right\rfloor + \lfloor x \rfloor^{\frac{3}{2}}\right) dx = \boxed{99000}$$

Integral 51

$$\int_1^e \left(\frac{\ln x}{x}\right)^2 \, dx$$

Integral 51 Answer

$$\int_1^e \left(\frac{\ln x}{x}\right)^2 \, dx = \boxed{2 - \frac{5}{e}}$$

Integral 52

$$\int_0^{400} \lfloor \sqrt{x} \rfloor \, dx$$

Integral 52 Answer

$$\int_0^{400} \lfloor \sqrt{x} \rfloor \, dx = \boxed{5130.75}$$