Mixer Round

Fall 2014 CHMMC

Fermi Questions

1. What is sin(1000)? (note: that's 1000 radians, not degrees)
2. In liters, what is the volume of 10 million US dollars' worth of gold?
3. How many trees are there on Earth?
4. How many prime numbers are there between 10^8 and 10^9?
5. What is the total amount of time spent by humans in spaceflight?
6. What is the global domestic product (total monetary value of all goods and services produced in a country's borders in a year) of Bangladesh in US dollars?
7. How much time does the average American spend eating during their lifetime, in hours?
8. How many CHMMC-related emails did the directors receive or send in the last month?

Suspiciously Familiar. . .

9. Suppose a farmer learns that he will die at the end of the year (day 365, where today is day 0) and that he has 100 sheep. He decides to sell all his sheep on one day, and that his utility is given by ab where a is the money he makes by selling the sheep (which always have a fixed price) and b is the number of days he has left to enjoy the profit; i.e., $365 - k$ where k is the day number. If every day his sheep breed and multiply their numbers by $(421 + b)/421$ (yes, there are small, fractional sheep), on which day should he sell out?
10. Suppose in your sock drawer of 14 socks there are 5 different colors and 3 different lengths present. One day, you decide you want to wear two socks that have *either* different colors *or* different lengths *but not both*. Given only this information, what is the maximum number of choices you might have?

I'm So Meta Even This Acronym

11. Let $\frac{s}{t}$ be the answer of problem 13, written in lowest terms. Let $\frac{p}{q}$ be the answer of problem 12, written in lowest terms.

    If player 1 wins in problem 11, let $n = q$. Otherwise, let $n = p$.

    Two players play a game on a connected graph with n vertices and t edges. On each player's turn, they remove one edge of the graph, and lose if this causes the graph to become disconnected. Which player (first or second) wins?

12. Let $\frac{s}{t}$ be the answer of problem 13, written in lowest terms.

    If player 1 wins in problem 11, let $n = t$. Otherwise, let $n = s$.

    Find the maximum value of

    $$\frac{x^n}{1 + \frac{1}{2}x + \frac{1}{4}x^2 + \ldots \frac{1}{2^{2n}}x^{2n}}$$

    for $x > 0$.

13. Let $\frac{p}{q}$ be the answer of problem 12, written in lowest terms.

    Let y be the largest integer such that 2^y divides p.

    If player 1 wins in problem 11, let $z = q$. Otherwise, let $z = p$.

    Suppose that $a_1 = 1$ and

    $$a_{n+1} = a_n - \frac{z}{n+2} + \frac{2z}{n+1} - \frac{z}{n}$$

    What is a_y?